

# OUTLIER ROBUST GMM ESTIMATION OF LEVERAGE DETERMINANTS IN LINEAR DYNAMIC PANEL DATA MODELS

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The GMM estimator usually employed in the panel data literature, has an unbounded influence function. This means that the estimator is easily influenced by outliers in the data. This paper develops a variant of the GMM estimator that is less sensitive to anomalous observations. Conditions for consistency and asymptotic normality of the robust estimator are presented. The robustness properties of the new estimator are investigated in a simulation experiment. An empirical illustration is provided, in which the determinants of a firm's capital structure are investigated using a panel of American firms. The application shows that the robust GMM estimator is a useful diagnostic tool for empirical econometric model building.

KEYWORDS: Robust estimation, dynamic panel data model, observation weighted GMM, aberrant observations, influence function, capital structure determinants.

## 1. INTRODUCTION

The increased availability of detailed microdata sets and better computer facilities have led to an increased popularity of panel data models in modern econometrics. Dynamic panel data models are of specific interest, as most decision processes in economics are of an intertemporal nature. It is nontrivial to estimate the parameters of dynamic panel data models consistently. For example, the maximum likelihood estimator generally produces inconsistent estimates if the number of periods in the panel remains constant, see, e.g., Nickell (1981) and Hsiao (1986). Several consistent estimators have been suggested, see, e.g., Anderson and Hsiao (1981), Blundell and Smith (1991), Arellano and Bond (1991), and Ahn and Schmidt (1995). Especially the approach of Arellano and Bond (1991) using the generalized method of moments (GMM) estimator of Hansen (1982) has gained widespread popularity. This estimator delivers consistent estimates under very weak distributional assumptions.

In applied work based on dynamic panel data models, one has to be aware of the possible adverse effects of aberrant observations on model estimation, inference, and interpretation. Such observations can have disastrous effects on standard parameter estimates if they are not properly accounted for, see Huber (1981), Hampel et al. (1986),

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and Rousseeuw and Leroy (1987). A prominent example is a panel data model where the moment restrictions defining the parameter estimates are violated by a small set of individuals. This can happen if, for example, the population consists of a large subpopulation satisfying the model conditions and a small subpopulation that displays a distinctly different behavior. Krasker et al. (1983, Section 1) convincingly argue that aberrant observations are more likely to appear in cross-sectional than in time series data. As for a typical panel the number of cross-sectional units is much larger than the number of periods, the possibility of aberrant observations seems particularly worrying.

In this paper we show that the standard GMM approach as adopted by Arellano and Bond (1991) and Ahn and Schmidt (1995) is very sensitive to aberrant observations. Therefore, we propose a different estimation method based on an outlier robust variant of GMM. We evaluate the standard and the new approach using asymptotic theory, simulations, and an empirical example.

It turns out that the outlier robust method constitutes a useful addition to the econometrician's tool-kit. The estimator provides automatic protection against aberrant observations by replacing the standard moment conditions by observation weighted moment conditions. As such, the robust estimator can be used as a diagnostic to check whether estimation results obtained with the traditional GMM estimator are driven by a few aberrant data structures. Moreover, the observation weights produced by the robust estimator can be used as a diagnostic device to assess which observations are not described by the postulated model. In this way, our robust method can give useful (additional) guidance for possible directions of model re-specification. The robust GMM estimator has similar asymptotic properties as the traditional GMM procedure. It is consistent and asymptotically normal at the usual rate.

Using outlier robust estimation comes at a certain price. If there are no aberrant observations, outlier robust GMM is often less efficient than standard GMM. This is intuitively clear, as the robust method assigns less weight to some of the observations. This results in a loss of efficiency if all observations are fully informative. The efficiency loss can be regarded as an insurance premium for being insured against the bad effects of aberrant observations. The height of the insurance premium (i.e., efficiency loss) and, accordingly, the level of protection (i.e., robustness), can be regulated by the user. The researcher can tune the robust approach in such a way that the robustness properties are adequate and the efficiency loss is only marginal, see the simulations in Section 4.

The robust GMM estimator is somewhat more computer intensive than its nonrobust counterpart. Moreover, the robust estimator is more flexible than its nonrobust counterpart: the practitioner has to make a few additional choices as to the specification of certain functions used in the robust procedure. The crucial choice, in fact, is to choose a central model in order to define discordant behavior. Deviations from this central model are classified as aberrant. Note that without such a central model, it seems impossible to talk about aberrant behavior at all, see Davies and Gather (1993). The present paper presents some default choices that can be used in applied work and that perform satisfactorily in the settings that are studied.

Before presenting the set-up of the paper, it is worthwhile to spend a few more lines

on the nature of aberrant observations. First, the occurrence of aberrant observations in statistical data sets is rather common, see Hampel et al. (1986). Econometricians and statisticians have long recognized this and taken measures to cope with such observations, e.g., by using preliminary data scanning procedures or by the post-estimation introduction of dummy variables. Second, aberrant observations need not be ‘bad’ observations. Their presence can be merely due to the approximate character of the model. Given the infinite complexity of reality and a postulated model of finite complexity, there may well be a set of observations that is not adequately captured by the model. Ideally, one would like to increase the complexity of the model in order to describe the aberrant observations as well as the bulk of the data. There are at least two reasons why such an increase in model complexity may be impossible. First, the outliers may not be detected, such that the researcher is unaware of the fact that particular (groups of) observations are not described by the model. Therefore, it is important that the estimation procedure gives a clear indication as to which observations are not described by the model, such that the researcher receives useful clues for directions of model augmentation and/or re-specification. Robust procedures are generally more suited for this purpose than nonrobust procedures, especially if aberrant observations occur in clusters, see Rousseeuw and Leroy (1987). Second, it is often impossible to increase the model’s complexity in the desired direction due to a lack of data. If only a few observations display aberrant behavior, it can hardly be expected that one can reliably identify variables or functional specifications that explain the behavior of these observations. This is clearly illustrated by the existence of (outlier) *robust non-parametric* techniques, e.g., Boente and Fraiman (1991) and Koenker et al. (1994): even if no a priori restrictions are placed on the functional specification of the model, one still has to worry about the possible adverse effect of aberrant observations.

If a model describing *all* the data cannot be constructed, a second best solution is to come up with a model that describes the bulk of the data, see also Krasker et al. (1983). This is the approach one usually encounters in econometrics, where the introduction of dummy variables and the use of preliminary data scanning procedures is common practice. The outlier robust GMM estimator of the present paper stands in this long tradition in that it proposes a *formal* and *automated* way to cope with aberrant observations.

The paper is set up as follows. Section 2 presents the model and the basic notation. In Section 3 we introduce the outlier robust GMM estimator. We present an asymptotic statistical theory for this estimator based on low-level assumptions, investigate its robustness properties, and give a detailed exposition of its implementation. In Section 4, we compare the performance of the standard and outlier robust version of the GMM estimator by means of a Monte-Carlo experiment. In Section 5, we apply both estimators to a panel of nonfinancial, American companies. This section illustrates the usefulness of the robust estimator for application oriented researchers. Section 6 concludes. The Appendix gathers the proofs of the theorems. More appendices are provided in the working paper version of this article, Lucas, van Dijk, and Kloek (1994), which is

available upon request.<sup>5</sup> These additional appendices contain a description of the data, algorithms for computing the estimator, further simulation and estimation results, and a generalization of the robust GMM estimator to the unbalanced panel case.

## 2. THE MODEL

Throughout this paper, we consider the linear dynamic panel data model

$$(1) \quad y_{it} = \alpha y_{i,t-1} + x'_{it}\beta + u_{it},$$

with  $i = 1, \dots, N$ ,  $t = 1, \dots, T$ , and  $N$  and  $T$  denoting the number of individuals and periods, respectively. We assume that  $T/N$  is small, such that we have a typical panel with many individuals that are observed during a few periods. Analogously, we only consider the semi-asymptotic case  $N \rightarrow \infty$  for  $T$  fixed when discussing consistency and asymptotic normality. The  $K$ -vector  $x_{it}$  contains the explanatory variables. In this article we assume throughout that the panel is complete. The working paper version presents some extensions for the case of incomplete panels.

We assume an error-components structure for  $u_{it}$ , such that  $u_{it}$  is the sum of an individual-specific, time-invariant term  $\mu_i$ , and an individual-specific, time-varying term  $\varepsilon_{it}$ ,  $u_{it} = \mu_i + \varepsilon_{it}$ . In this way we allow the means of  $u_{it}$  to differ over individuals. By defining  $z'_{it} = (y_{i,t-1}, x'_{it})$  and  $\gamma' = (\alpha, \beta')$ , (1) can be rewritten as

$$(2) \quad y_{it} = z'_{it}\gamma + \mu_i + \varepsilon_{it}.$$

It will prove useful to introduce the following notation. Let  $y'_i = (y_{i1}, \dots, y_{iT})$  and  $y' = (y'_1, \dots, y'_N)$ . Similarly, let  $Z'_i = (z_{i1}, \dots, z_{iT})$  and  $Z' = (Z'_1, \dots, Z'_N)$ . The vectors  $\varepsilon_i$  and  $\varepsilon$  and the matrices  $X_i$  and  $X$  are defined analogously to  $y_i$ ,  $y$ ,  $Z_i$ , and  $Z$ , respectively. Furthermore, let  $D_T$  be a  $(T-1) \times T$  matrix, which is defined as

$$(3) \quad D_T = \begin{pmatrix} -1 & 1 & 0 & \cdot & 0 & 0 \\ 0 & -1 & 1 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & -1 & 1 \end{pmatrix}.$$

$D_T$  transforms a vector  $y_i$  in a vector of first differences,  $D_T y_i = (\Delta y_{i2}, \dots, \Delta y_{iT})'$ , with  $\Delta$  the first difference operator,  $\Delta y_{it} = y_{it} - y_{i,t-1}$ . Using these definitions, (2) can be differenced and rewritten as

$$(4) \quad D_T y_i = D_T Z_i \gamma + D_T \varepsilon_i.$$

In Section 3 we introduce orthogonality (or moment) conditions to estimate the parameters in (4). In order to define these conditions, it is helpful to introduce the

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<sup>5</sup> The paper may also be obtained on the World Wide Web at

<http://www.econ.vu.nl/vakgroep/bfs/alm/lucas/papers/gmm.abs>

This address can also be used to download ANSI-C sources used for the calculations in this paper.

matrix

$$(5) \quad W_i = \begin{pmatrix} y_i^0 & 0 & \cdots & 0 & x_i^T & 0 & \cdots & 0 \\ 0 & y_i^1 & \cdots & 0 & 0 & x_i^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & y_i^{T-2} & 0 & 0 & \cdots & x_i^T \end{pmatrix},$$

with  $y_i^t = (y_{i0}, \dots, y_{it})$ , and  $x_i^T = (\text{vec}(X_i'))'$ . The matrix in (5) is taken from Ahn and Schmidt (1995) and differs from the instrument matrix used by Arellano and Bond (1991), because (5) is based on a strict rather than a weak exogeneity assumption for the  $x_{it}$  variables. If this assumption is correct, using the large  $W_i$  matrix in (5) results in an asymptotically more efficient estimator, see Ahn and Schmidt (1995). This efficiency gain extends to finite samples, as is demonstrated by Breitung (1994).

### 3. OUTLIER ROBUST GMM

In this section we introduce the outlier robust GMM estimator. In Subsection 3.1 we present the GMM estimator and the specific moment conditions used for parameter estimation. Subsection 3.2 gives some details on the implementation of the estimator. Subsection 3.3 provides the asymptotic theory for the outlier robust GMM estimator. Subsection 3.4 evaluates the asymptotic robustness properties of the estimator and, as a side result, demonstrates the nonrobustness of the standard GMM estimator. The moment conditions and the outlier robust estimator proposed in this section can be used for a general class of linear models and linear moment restrictions. To fix ideas, however, we use the example of the GMM estimator for linear dynamic panel data models to illustrate some of the properties of the robust procedure.

#### 3.1. Observation Weighted Moment Conditions

A first step in the construction of the outlier robust GMM estimator consists of setting up appropriate moment conditions that identify the parameters. Apart from more technical regularity conditions mentioned in the Appendix, the main assumption we make is the following.

**Assumption 1.** *Conditional on some information set  $I_{it}$ ,  $\varepsilon_{it}$  and  $\varepsilon_{i,t+1}$  are independently and identically distributed.*

Assumption 1 mimics the exchangeability assumption made by Honoré (1992) in the context of truncated and censored regression models for panel data. If  $I_{it} = \{\mu_i, x_{i1}, \dots, x_{iT}, y_{i0}, \dots, y_{i,t-1}\}$ , Assumption 1 implies the moment conditions presented in Ahn and Schmidt (1995). Alternatively, if one uses  $I_{it} = \{\mu_i, x_{i1}, \dots, x_{it}, y_{i0}, \dots, y_{i,t-1}\}$ , Assumption 1 implies the moment conditions of Arellano and Bond (1991). Assumption 1 is somewhat more restrictive than the assumptions in Ahn and Schmidt (1995), which only require uncorrelatedness instead of independence. Note, however, that Assumption 1 still allows for a considerable degree of heterogeneity across individuals, e.g., different

error variances across individuals. In Section 4 we discuss the effect of violations of Assumption 1 on our estimation procedures.

The main implication of Assumption 1 is that the conditional density of  $\varepsilon_{i,t+1} - \varepsilon_{it}$  is symmetric around zero, where the conditioning is with respect to  $I_{it}$ . This implication lies at the basis of the subsequent asymptotic analysis.

The main principle used in the outlier robust GMM estimator is that we replace standard moment conditions by a set of observation weighted moment conditions. In order to specify the weights for the robust GMM estimator, we need the following definitions. Let  $\Phi_{iN}$  denote a diagonal matrix, with the  $t$ th diagonal element equal to  $\phi_{itN}$ . The quantity  $\phi_{itN}$  can be interpreted as the weight for individual  $i$  at time  $t$ . The dependence on  $N$  stems from the fact that the weights are based on estimates of location and scale measures, see the remainder of this subsection. Note that  $\phi_{itN}$  is available after estimation and can be used as a diagnostic measure for finding aberrant observations with respect to the postulated model, cf. Franses and Lucas (1998). Let  $w_{it}$  denote a vector of instrumental variables to be used in the moment condition for individual  $i$  at time  $t$ , e.g., the  $t$ th column of the matrix  $W'_i$  in (5). Furthermore, let  $e_{it} = e_{it}(\gamma) = \Delta y_{i,t+1} - \Delta z'_{i,t+1}\gamma$ . We impose the following structure on  $\phi_{itN}$ :

$$(6) \quad \phi_{itN} = v_{tN}(w_{it}) \cdot \hat{\sigma}_N \psi(e_{it}/\hat{\sigma}_N)/e_{it},$$

if  $e_{it} \neq 0$ , and  $\phi_{itN} = v_{tN}(w_{it})$  otherwise. In this definition,  $\psi(\cdot)$  and  $v_{tN}(\cdot)$  are real valued functions. More details on possible specifications of these functions are given in Subsection 3.2. The scalar  $\hat{\sigma}_N$  is a scale equivariant estimate of the scale  $\sigma$  of  $e_{it}$ . The parameters  $\sigma$  and  $\gamma$  are estimated iteratively.

As can be seen from (6), the weight factor  $\phi_{itN}$  can be split into two parts. The first part depends on  $w_{it}$  through the function  $v_{tN}$ . This part is used to reduce the effect of discordant or influential observations in the space of instrumental variables. The remaining factor depends on  $e_{it}$  and serves to downweight observations with large residuals. If the function  $\psi$  is bounded, it is easy to see that observations with a large absolute value for  $e_{it}$  receive a small weight. The present two components structure of  $\phi_{itN}$  follows the weighting scheme proposed by Mallows in the ordinary regression setting, see Hampel et al. (1986, page 321). Note that the standard GMM estimator of, e.g., Ahn and Schmidt (1995), assigns unit weights to all observations. It can be obtained as a special case of (8) by setting  $\psi(e_{it}/\hat{\sigma}_N) = e_{it}/\hat{\sigma}_N$  and  $v_{tN}(w_{it}) \equiv 1$ .

Given the above definition of the weights  $\phi_{itN}$ , we can now present the moment conditions identifying the unknown parameters. This is stated in the following lemma, which is proved in the Appendix.

**LEMMA 1:** *Let  $\gamma_0$  denote the true parameter vector for model (2), and define  $e_{it}^0 = e_{it}(\gamma_0)$ . Given Assumption 1, a positive constant  $\sigma$ , an anti-symmetric function  $\psi(\cdot)$  ( $\psi(-e) = -\psi(e)$ ), and  $w_{it} \in I_{it}$ , we have*

$$(7) \quad E \left( \sum_{t=1}^{T-1} w_{it} \cdot \phi_{it} \cdot e_{it}^0 \right) = 0,$$

assuming these expectations exist, where

$$\phi_{it} = v_t(w_{it}) \cdot \sigma \psi(e_{it}^0/\sigma)/e_{it}^0$$

for  $e_{it}^0 \neq 0$ , and  $\phi_{it} = v_t(w_{it})$  otherwise, with  $v_t(\cdot)$  denoting a real valued function.

Lemma 1 implies that a conditional independence assumption combined with a function  $\psi(\cdot)$  that is anti-symmetric around zero produces a set of moment conditions that can be used to estimate the parameters in models such as (2). An alternative to the low-level conditions in Assumption 1 would be to impose the moment conditions in Lemma 1 directly as a type of high-level condition. The present set of low-level conditions, however, is much more insightful.

Two remarks are in order. First, as mentioned earlier, the conditions in Assumption 1 are stronger than those in Ahn and Schmidt (1995). Therefore, it is useful to study the performance of our robust procedure in case Assumption 1 is violated, while the conditions of Ahn and Schmidt (1995) are still valid. This is done using simulations in Section 4. Second, if the regression errors are heteroskedastic in the cross sectional direction and if the function  $\psi(\cdot)$  is bounded, then the moment conditions in Lemma 1 make use of an automated GLS type weighting: large residuals (caused by a large value of the individual variance) receive less weight due to boundedness of  $\psi(\cdot)$ . This would also be the case if the observations were to be weighted by the inverses of their (estimated) individual standard errors, such that the use of a robust estimator can cause an efficiency improvement under cross-sectional heteroskedasticity.

If we consider the special case of estimating dynamic panel data models, we see that by an appropriate choice of  $\phi_{it}$  the moment conditions in (7) reduce to the standard moment conditions as used in, e.g., Ahn and Schmidt (1995). Define  $W_i'$  as a matrix with column  $t$  equal to  $w_{it}$ ,  $\Phi_i$  a diagonal matrix with  $t$ th element  $\phi_{it}$ , and  $e_i^0$  a vector with  $t$ th element  $e_{it}^0$ , then the moment conditions in Lemma 1 can be rewritten as  $E(W_i' \Phi_i e_i^0) = 0$ . From this it can be seen that (7) reduces to the moment conditions in Ahn and Schmidt (1995) if  $\Phi_i \equiv I$ .

Based on Hansen (1982) and the moment conditions in (7), we define the (outlier robust) GMM estimator as

$$\hat{\gamma}_N = \arg \min_{\gamma} \left[ N^{-1} \sum_{i=1}^N W_i' \Phi_{iN} D_T(y_i - Z_i \gamma) \right]' \cdot A_N \cdot \left[ N^{-1} \sum_{i=1}^N W_i' \Phi_{iN} D_T(y_i - Z_i \gamma) \right], \quad (8)$$

with  $\{A_N\}_{N=1}^{\infty}$  a sequence of matrices that converges almost surely to some positive definite matrix  $A_0$ .

### 3.2. Implementation of the Estimator

In order to make the outlier robust GMM estimator operational, we need to specify the functions  $\psi(\cdot)$  and  $v_{tN}(\cdot)$  in (6). In choosing the functional form of the weight functions,

we are primarily led by the objective of *protection* against aberrant observations. Our second concern is that of the efficiency of the estimator.

Several alternative specifications of weight functions are available in the literature, see Huber (1981) and Hampel et al. (1986). In the present paper we use a specification for  $\psi$  that produces weights that can easily be interpreted, namely

$$(9) \quad \psi(e) = \begin{cases} e & \text{if } |e| \leq c_1, \\ \tilde{\psi}(|e|) \cdot \text{sgn}(e) & \text{if } c_1 < |e| \leq c_2, \\ 0 & \text{if } |e| > c_2, \end{cases}$$

where  $0 < c_1 < c_2$ ,  $\text{sgn}(\cdot)$  is the sign function, and where  $\tilde{\psi}(|e|)$  is a fifth degree polynomial in  $|e|$  such that  $\psi(e)$  is twice continuously differentiable. The  $\psi(\cdot)$  function in (9) resembles that of Campbell (1980) in shape, but the function of Campbell is not differentiable, whereas (9) has two continuous derivatives. Using (9), observations with standardized absolute residuals below  $c_1$  receive weight one, while observations with standardized residuals outside the range  $[-c_2, c_2]$  receive zero weight. The remaining observations are considered as borderline cases and are partially taken into account when determining the parameter estimates. The present specification combines the advantages of several well-known choices for  $\psi(\cdot)$  available in the literature, e.g., the Huber and the bisquare function, see Hampel et al. (1986): the implied weights can be easily interpreted, extreme outliers are automatically removed from the sample, and the function is sufficiently smooth for asymptotic analyses. Naturally, more traditional alternative specifications of  $\psi(\cdot)$  can also be used and yield procedures with slightly different properties. The present choice of  $\psi(\cdot)$ , however, appears adequate for empirical analysis of the type considered in this paper. Note that  $\psi$  in (9) is an anti-symmetric function, as required in Lemma 1.

The above specification of  $\psi$  requires the values of two tuning constants  $c_1$  and  $c_2$ . These constants determine both the outlier sensitivity and the efficiency of the estimator. A natural trade-off exists between these two. On the one hand, we want an estimator that is reasonably efficient if the sample contains no outliers. For the specification of  $\psi$  given above, this amounts to letting  $c_1, c_2 \rightarrow \infty$ . On the other hand, we want the estimator to be insensitive to outliers. This leads to setting  $c_1$  and  $c_2$  to some small, positive value. In this paper we set  $c_1^2 = \chi_1^{-2}(0.990)$  and  $c_2^2 = \chi_1^{-2}(0.999)$ , where  $\chi_p^{-2}(\cdot)$  is the inverse cumulative distribution function (c.d.f.) of the  $\chi^2$  distribution with  $p$  degrees of freedom. This implies that we define discordancy with respect to (2) for Gaussian  $e_{it}$ . Values of  $e_{it}$  occurring with a probability of less than 0.0005 under a Gaussian distribution are discarded, while those occurring with a probability between 0.0005 and 0.005 are partially downweighted. All remaining observations are fully taken into account. As a result, the robust GMM estimator will be highly efficient if the error distribution is close to the Gaussian distribution. Moreover, the estimator will be outlier robust, because extreme observations are automatically removed from the sample. Alternative values of  $c_1$  and  $c_2$  are also possible and give rise to a different trade-off between efficiency and robustness.

As an estimator  $\hat{\sigma}_N$  for  $\sigma$  we use the scaled median absolute deviation,

$$(10) \quad \hat{\sigma}_N = 1.483 \cdot \text{median}_{i,t} |\hat{e}_{it} - \text{median}_{i,t}(\hat{e}_{it})|,$$

with  $\hat{e}_{it} = e_{it}(\hat{\gamma})$  for some estimator  $\hat{\gamma}$  of  $\gamma_0$ . The scaling makes  $\hat{\sigma}_N$  a consistent estimator of the standard deviation under the assumption of normality. As an alternative, the scaling could be omitted and incorporated in the appropriate choice of the tuning constants  $c_1$  and  $c_2$ . Note that  $\hat{\sigma}_N$  converges almost surely to a nonzero constant if  $\hat{\gamma}$  is strongly consistent and if the distribution of  $e_{it}^0$  is sufficiently regular.

To complete the description of the robust GMM estimator, we have to specify the weight function  $v_{tN}$ . If the estimator is to be less sensitive to outliers,  $v_{tN}(w_{it})$  must become small if  $w_{it}$  is far removed from the bulk of  $w_{it}$ -vectors. In order to judge whether the latter is the case, we use the Mahalanobis distance of  $w_{it}$ , which is defined as

$$(11) \quad \delta(w_{it}, m_t, V_t) = \sqrt{(w_{it} - m_t)' V_t^{-1} (w_{it} - m_t)},$$

with  $m_t$  and  $V_t$  representing the location vector and covariance matrix of  $w_{it}$ , respectively. Define  $\delta_{it} = \delta(w_{it}, m_t, V_t)$  and  $\hat{\delta}_{itN} = \delta(w_{it}, \hat{m}_{tN}, \hat{V}_{tN})$ , with  $\hat{m}_{tN}$  and  $\hat{V}_{tN}$  estimates of  $m_t$  and  $V_t$ , respectively. If  $V_t$  is singular, a pseudo inverse has to be used in (11). Using the Mahalanobis distance, we set  $v_{tN}(w_{it}) = \psi_t(\hat{\delta}_{it}^2)/\hat{\delta}_{it}^2$  if  $\hat{\delta}_{it} \neq 0$ , and  $v_{tN}(w_{it}) = 1$  otherwise, where  $\psi_t$  is a function such that aberrant  $w_{it}$  vectors receive a smaller weight. In the present paper, we specify  $\psi_t$  as in (9) with tuning constants  $c_{1t}$  and  $c_{2t}$ . These tuning constants are specified as  $c_{1t} = \chi_{p_t}^{-2}(0.990)$  and  $c_{2t} = \chi_{p_t}^{-2}(0.999)$ , respectively, with  $p_t = \text{rank}(V_{tN})$ . The tuning constants may vary with  $t$  because the number of available moment conditions may vary with the period considered. This is clearly illustrated by the different columns of  $W_i'$  in (5), which contain a different number of non-zero elements. The choice for  $\chi^2$ -based tuning constants again implies that we define discordancy with respect to a multivariate normal distribution for the  $w_{it}$  vectors. Other values of the tuning constants (implying different central or outlier free models) could also be used, yielding a different performance.

It is not advisable to use the sample mean and the traditional covariance matrix estimator,  $\hat{V}_{tN} = (N-1)^{-1} \sum_{i=1}^N (w_{it} - \hat{m}_{tN})(w_{it} - \hat{m}_{tN})'$ , as estimates of  $\hat{m}_{tN}$  and  $\hat{V}_{tN}$ , respectively, as these estimators are sensitive to aberrant observations, see Hampel et al. (1986) and Rousseeuw and Leroy (1987). In this paper we use the S estimator for  $(\hat{m}_{tN}, \hat{V}_{tN})$  as described in Lopuhaä (1989) and Lopuhaä and Rousseeuw (1991). We use the projection algorithm of Rousseeuw and van Zomeren (1990) to approximate the estimator. The S estimator is an affine equivariant<sup>6</sup> estimator of location and covariance. Formally, it is defined as the combination  $(\hat{m}_{tN}, \hat{V}_{tN})$  that minimizes  $|\hat{V}_{tN}|$  subject to

$$(12) \quad N^{-1} \sum_{i=1}^N \rho\{[(w_{it} - \hat{m}_{tN})' \hat{V}_{tN}^{-1} (w_{it} - \hat{m}_{tN})]^{1/2}\} \geq b,$$

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<sup>6</sup>For a definition of affine equivariance, see Rousseeuw and Leroy (1987, page 250).

with  $\rho$  a bounded function and  $b$  a positive, real constant. A typical value for  $b$  is  $0.5 \sup_e \rho(e)$ . Following the suggestion in Lopuhaä (1989), we define  $\rho(e)$  as

$$\rho(e) = \begin{cases} (c^2/6) \cdot (1 - (1 - (e/c)^2)^3) & \text{if } |e| \leq c, \\ c^2/6 & \text{otherwise.} \end{cases}$$

The derivative of  $\rho(e)$  is the bisquare function  $\psi(e)$  of Beaton and Tukey (1974). Note that the function  $\rho$  requires the specification of a constant  $c$ . Informally stated, the S estimator minimizes  $|\hat{V}_{tN}|$  subject to the restriction that the ellipsoid

$$\{w \mid (w_{it} - \hat{m}_{tN})' \hat{V}_{tN}^{-1} (w_{it} - \hat{m}_{tN}) \leq k(c)\}$$

for a particular constant  $k(c)$ , covers a large enough number of observations  $w_{it}$ ,  $i = 1, \dots, N$ . In the working paper version of this article, we obtained a response surface for the constant  $c$  as a function of  $P = \text{rank}(V_{tN})$ . This response surface can be approximated by

$$(13) \quad c(P) = \begin{cases} 2.7851P^{1/2} - 1.1213 - 0.0903P & \text{for } 1 \leq P \leq 10, \\ 2.2135P^{1/2} - 0.1868 & \text{for } 10 < P. \end{cases}$$

These values of  $c(P)$  can be used in empirical work. Lopuhaä and Rousseeuw (1991) prove that the S estimator as described above can deal with data that contain up to 50% outliers. This is the highest possible percentage of contamination that affine equivariant estimators can resist to. Moreover, Davies (1987) and Lopuhaä (1989) show that under some weak regularity conditions, the S estimator is  $N^{1/2}$ -consistent and asymptotically normal.

As we need estimates of the location vector and covariance matrix for each  $t$ , it is computationally inefficient to calculate the (computer intensive) S estimator for each set of  $\{w_{it}\}_{i=1}^N$  for  $t = 1, \dots, T-1$ . Therefore, we describe a method for reducing the computational burden considerably by exploiting the linear structure of the elements of  $W_i$ . We illustrate the idea by considering the instrument matrix of Ahn and Schmidt (1995). The technique itself, however, is more general and can be applied to any  $W_i$  that has a linear structure.

Define the vector  $p_i = (y_i^T, x_i^T)'$ , with  $y_i^T$  and  $x_i^T$  as defined below (5). This vector contains all observations for individual  $i$ . Using the  $N$  observed  $p_i$  vectors and the S estimator, we estimate the location  $m$  and covariance  $V$  of  $p_i$  by  $\hat{m}_N$  and  $\hat{V}_N$ , respectively. Next, we exploit the fact that for the instrument matrix  $W_i$  in (5),  $w_{it}$  is a linear transformation of  $p_i$  for every value of  $t$ . In particular, for every  $t$  there is a matrix  $H_t$  such that  $w_{it} = H_t p_i$ . We use this linearity property to estimate the location and covariance of  $w_{it}$  by  $\hat{m}_{tN} = H_t \hat{m}_N$  and  $\hat{V}_{tN} = H_t \hat{V}_N H_t'$ , respectively. Note that  $(\hat{m}_{tN}, \hat{V}_{tN})$  is not necessarily an S estimator. This follows from the fact that the matrix  $H_t$  is not a square matrix, and, consequently, premultiplication by  $H_t$  is not an affine transformation. The robustness properties and speed of convergence of the S estimator  $(\hat{m}_N, \hat{V}_N)$  are, however, inherited by  $(\hat{m}_{tN}, \hat{V}_{tN})$ .

### 3.3. Consistency and Asymptotic Normality

As was mentioned in Section 2, we only consider the semi-asymptotic case  $N \rightarrow \infty$  for fixed  $T$ . Apart from Assumption 1, we assume that model (1) is correctly specified with true parameter value  $\gamma_0$  and that we may freely interchange the order of integration and differentiation. Before presenting the asymptotic distribution of the estimator, we briefly comment on the four major additional assumptions we make.

First, we assume that the vectors  $p_i$  are i.i.d. with common mean  $m$  and covariance matrix  $V$ . The *identically* part of this assumption can be relaxed as long as  $\hat{m}_N$  and  $\hat{V}_N$  have finite limits (at rate  $N^{1/2}$ ) and the limit of  $\hat{V}_N$  is positive definite. In this way we can allow for a considerable degree of heterogeneity across individuals. Note that even under the i.i.d. assumption for  $p_i$ , the elements of  $p_i$  may exhibit certain forms of dependence over *time*. Because the S-estimator for  $\hat{m}_N$  and  $\hat{V}_N$  is not the key issue in the present paper, we refrain from generalizing the result of Davies (1992) on the  $N^{1/2}$ -consistency of S-estimators to the case of independently, non-identically distributed  $p_i$ , and concentrate on the i.i.d. case. For a generalization to the non-i.i.d. case, see Sakata and White (1996).

Second, the functions  $\psi_t$  and  $\psi$  that are used in the construction of the weights, must be once and twice continuously differentiable, respectively. This condition can be relaxed if we are willing to assume that  $p_i$  has a sufficiently smooth density. In that case, one can allow for a countable number of discontinuity points in  $\psi$  and  $\psi_t$ , compare Hampel et al. (1986, p. 102).

Third, we assume that the parameters  $(\gamma, \sigma, m, V)$  are contained in a compact set. This assumption is fairly standard in the literature, see, e.g., Gallant (1987, Chapter 3) and the references cited therein. We also assume that the sequences  $\{\hat{\sigma}_N\}$  and  $\{\hat{A}_N\}$  converge almost surely and at the appropriate rate to  $\sigma$  and  $A_0$ , respectively, with  $A_0$  a positive definite matrix. A simple example is given by a nonoptimal GMM estimator  $A_N \equiv I$  and  $\hat{\sigma}_N$  the median absolute deviation.

Fourth, we require certain matrices and vectors to be finite and/or nonsingular, see the Appendix. We can now state the main theorem.

**THEOREM 1:** *Given the correctly specified model (2) with true parameter  $\gamma_0$ , given the conditions of Lemma 1, and given Assumptions 1 and 2 (stated in the Appendix), the robust GMM estimator defined in (8) is consistent. Moreover,  $\hat{\gamma}_N$  is asymptotically normally distributed:*

$$N^{1/2}(\hat{\gamma}_N - \gamma_0) \xrightarrow{d} N(0, M \cdot M_2 \cdot M'),$$

with

$$\begin{aligned} M &= (M_1' A_0 M_1)^{-1} M_1' A_0, \\ M_1 &= E \left( \sum_{t=1}^{T-1} w_{it} v_t(w_{it}) \psi'(e_{it}^0 / \sigma) \Delta z'_{i,t+1} \right), \end{aligned}$$

$$M_2 = E \left( \left( \sum_{t=1}^{T-1} \sigma w_{it} v_t(w_{it}) \psi(e_{it}^0/\sigma) \right) \left( \sum_{t=1}^{T-1} \sigma w_{it} v_t(w_{it}) \psi(e_{it}^0/\sigma) \right)' \right),$$

$$v_t(w_{it}) = \begin{cases} \psi_t(\delta_{it}^2)/\delta_{it}^2 & \text{if } \delta_{it} \neq 0, \\ 1 & \text{otherwise,} \end{cases}$$

$\delta_{it}$  as defined below (11),  $e_{it}^0 = e_{it}(\gamma_0)$ , and  $\psi'(e_{it}^0) = d\psi(e_{it}^0)/de_{it}^0$ .

**Remark:** The uncertainty associated with the estimation of the scale parameter  $\sigma$  and of the location vector and scaling matrix of the  $w_{it}$  vectors does not enter the limiting distribution of  $\hat{\gamma}_N$ . This is a result of the anti-symmetry and boundedness of  $\psi(\cdot)$ , see the proof in Appendix B. Consequently, the information matrix is block-diagonal. Also note that certain moment conditions are satisfied automatically because of the weight functions that are used for the outlier robust GMM estimator.

The asymptotic covariance matrix of the robust GMM estimator,  $MM_2M'$ , depends upon the matrix  $A_0$ . Given the moment conditions in (7), the optimal choice for  $A_0$  from a minimum variance perspective is  $A_0 = M_2^{-1}$ . The optimal robust GMM estimator can be computed with the familiar two-step approach. In the first step, robust GMM estimates are computed using a non-optimal choice for  $A_N^{(0)}$ . The residuals obtained in this first step are used to estimate  $M_2$  using the definition in Theorem 1, replacing expectations by averages, see Hansen (1982). The optimal robust GMM estimator is obtained by setting  $A_N^{(1)}$  equal to the inverse of the estimate of  $M_2$  and recalculating the estimator. Note that using this two-step procedure requires the estimates of the first step to be reasonable starting values for the second step. If the preliminary estimates are completely incorrect, all residuals might become very large, resulting in zero weights for all observations using the specification of  $\psi(\cdot)$  from Subsection 3.2. Consequently,  $M_2$  might implode to a singular matrix in this case. This situation, however, can be controlled for by considering alternative starting values in the minimization procedure.

To conclude this subsection, we comment on the relation between our robust estimator and the least absolute deviations (LAD) estimator, as, e.g., proposed for the censored regression model with individual effects in Honoré (1992). Using similar techniques as in Hampel et al. (1986, Chapter 2) and abstracting from the complications due to censoring, one easily establishes that Theorem 1 produces a similar result as Honoré (1992). Compared to the standard GMM procedure, the LAD-based GMM estimator assigns smaller weights to large residuals. In that sense it can be called robust. The principal difference with the robust estimator proposed in the present paper, however, is that outliers in the space of instrument vectors ( $w_{it}$ ) are not accounted for. As a result, the LAD estimator is still extremely vulnerable to certain types of outliers, see Hampel et al. (1986, Chapter 6).

### 3.4. Influence Function

In order to assess the robustness properties of the robust GMM estimator, we consider the concept of the influence function (IF). The IF measures the effect on an estimator of small perturbations in the distribution of the observations. Informally, the IF measures the effect on the estimator of adding a very small fraction of arbitrary outliers to the data. In order to derive the IF, we assume throughout that the conditions of Theorem 1 are met.

The IF is an asymptotic concept. Therefore, we first present the functional representation of the robust GMM estimator. Let the distribution function of  $p_i$  be given by  $F_\eta = (1 - \eta)F_0 + \eta G$ .  $F_0$  is the central, uncontaminated distribution,  $G$  is the distribution that generates the outliers, and  $\eta \in [0, 1]$  is the fraction of outliers. Using the moment conditions in (7), the robust GMM estimator  $\hat{\gamma}(F_\eta)$  is defined as

$$(14) \quad \hat{\gamma}(F_\eta) = \arg \min_{\gamma} E_\eta(W_i' \Phi_i D_T(y_i - Z_i \gamma))' A_0 E_\eta(W_i' \Phi_i D_T(y_i - Z_i \gamma)),$$

with  $E_\eta$  denoting the expectations operator with respect to the distribution  $F_\eta$ , and where we assume that  $\hat{\gamma}(F_\eta)$  is uniquely defined. The IF is given by

$$IF(G, \hat{\gamma}, F_0) = \lim_{\eta \downarrow 0} \frac{\hat{\gamma}(F_\eta) - \hat{\gamma}(F_0)}{\eta}$$

whenever this limit exists. Note that the IF is a directional derivative of  $\hat{\gamma}$  at  $F_0$  in the direction  $G$ , i.e.,  $IF(G, \hat{\gamma}, F_0) = d\hat{\gamma}(F_\eta)/d\eta|_{\eta=0}$ . In order to simplify the notation, we introduce the matrix  $B_\eta$  and the vector  $C_\eta$ :

$$B_\eta = E_\eta \left( \sum_{t=1}^{T-1} w_{it} v_t(w_{it}) \psi'(e_{it}^\eta / \sigma) \Delta z'_{i,t+1} \right)$$

and

$$C_\eta = E_\eta \left( \sum_{t=1}^{T-1} \sigma w_{it} v_t(w_{it}) \psi(e_{it}^\eta / \sigma) \right),$$

with  $e_{it}^\eta = e_{it}(\hat{\gamma}(F_\eta))$ . Differentiating the right-hand side of (14) with respect to  $\gamma$ , we obtain the first order condition

$$(15) \quad B_\eta A_0 C_\eta = 0.$$

Differentiating both sides of (15) with respect to  $\eta$  and evaluating the result at  $\eta = 0$ , we obtain

$$(16) \quad \left. \frac{dB'_\eta}{d\eta} \right|_{\eta=0} A_0 C_0 - B'_0 A_0 B_0 \cdot IF(G, \hat{\gamma}, F_0) + B'_0 A_0 E_G \left( \sum_{t=1}^{T-1} \sigma w_{it} v_t(w_{it}) \psi(e_{it}^0 / \sigma) \right) = 0,$$

with  $E_G$  denoting the expectations operator with respect to the distribution  $G$ . Because of (7),  $C_0$  is equal to zero. Furthermore, we assume that the derivative of  $B_\eta$  with respect

to  $\eta$  is finite at  $\eta = 0$ . This trivially holds for the choices of  $\psi$  and  $\psi_t$  given in Subsection 3.2. Equation (16) can now be rewritten as

$$(17) \quad IF(G, \hat{\gamma}, F_0) = (B_0' A_0 B_0)^{-1} B_0' A_0 E_G \left( \sum_{t=1}^{T-1} \sigma w_{it} v_t(w_{it}) \psi(e_{it}^0 / \sigma) \right).$$

Given the specifications of  $\psi(\cdot)$  and  $v_t(\cdot)$ , the IF of the robust GMM estimator is clearly bounded due to the fact that aberrant observations receive smaller weights. Large residuals are kept within bounds by the function  $\psi$ , while anomalous instrument vectors  $w_{it}$  are downweighted by the function  $v_t$ . For the standard GMM estimator we have  $v_t(\cdot) \equiv 1$  and  $\psi(e) = e$ , such that (17) also clearly illustrates that the IF of this standard GMM estimator is unbounded. Consequently, outliers can have a large influence on standard nonrobust GMM.

#### 4. SIMULATION RESULTS

In this section we investigate the properties of the robust GMM estimator using a limited Monte-Carlo study. We study a special case of the robust GMM estimator as presented in Section 3, namely the estimator of Ahn and Schmidt (1995) for linear dynamic panel data models with individual effects. When simulating such models, it is important to limit the number of free parameters. We adopt the elegant simulation set-up of Kiviet (1995) to achieve this objective.

The model for generating the data is a linear, dynamic panel data model with one exogenous variable:

$$(18) \quad \begin{aligned} v_{it} &= \alpha v_{i,t-1} + \beta x_{it} + \varepsilon_{it}, \\ x_{it} &= \rho x_{i,t-1} + \xi_{it}, \\ y_{it} &= v_{it} + \mu_i / \beta, \end{aligned}$$

where  $\varepsilon_{it}$ ,  $\xi_{it}$ , and the individual effect  $\mu_i$  are i.i.d. normal random variables with zero means and variances 1,  $\sigma_\xi^2$ , and  $\beta^2$ , respectively. In order to let the system be stationary, we use the technique of Kiviet to choose appropriate values for  $x_{i0}$ ,  $y_{i0}$  and  $y_{i1}$ . The variance parameter  $\sigma_\xi^2$  equals

$$\beta^{-2} \left( \sigma_*^2 - \frac{\alpha}{1 - \alpha^2} \right) \left( 1 + \frac{(\alpha + \rho)^2}{1 + \alpha\rho} (\alpha\rho - 1) - (\alpha\rho)^2 \right),$$

with  $\sigma_*^2$  the variance of  $v_{it} - \varepsilon_{it}$ , i.e., the signal-to-noise ratio. Following Kiviet, we impose the restriction  $\alpha + \beta = 1$ . There are now three free parameters left, namely  $\alpha$ ,  $\rho$ , and  $\sigma_*^2$ . We set  $\rho = 0.8$  and let  $\alpha$  equal 0.0, 0.4, or 0.8. For  $\sigma_*^2$  we choose the values 2 and 8. These parameter values are also used by Kiviet in his simulation study. Furthermore, we set  $N = 100$  and  $T = 6$ .

In order to compute the robust GMM estimator, we use the functional forms for  $\psi$  and  $\psi_t$  presented in Subsections 3.2. The nonrobust GMM estimator is computed with  $\psi(\cdot)$  and  $\psi_t(\cdot)$  both being equal to the identity function.

As an estimator for the scale  $\sigma$  of  $e_{it}$ , we use the scaled median absolute deviation as in (10). This scale estimate is updated simultaneously with the remaining unknown parameters. The matrix  $A_N$  is computed using the optimal two-step procedure described in Subsection 3.3. The moment weighting matrix that is used for the first-step consistent estimator is the inverse of

$$(19) \quad N^{-1} \sum_{i=1}^N W_i' \tilde{\Phi}_{iN} D_T D_T' \tilde{\Phi}_{iN} W_i,$$

where  $\tilde{\Phi}_{iN}$  is a diagonal matrix with the  $t$ th diagonal element being equal to  $v_{tN}(w_{it})$ . Note that  $D_T D_T'$  is proportional to  $E(D_T \varepsilon_i \varepsilon_i' D_T')$  if  $\varepsilon_i$  has a scalar covariance matrix  $\sigma_\varepsilon^2 I$ .

In order to reveal the differences between the robust and the nonrobust GMM estimator, we also compute the estimators for samples with outliers. We use the following outlier generating mechanism. For a generated panel, we compute the median absolute deviations of all variables over all individuals and periods. Next, we randomly select 5% of the individuals or cross-sectional units. For each of these cross-sectional units, we add/subtract (with equal probability) two times the appropriate median absolute deviation to/from every observation. Consequently, the outliers satisfy two criteria: (1) they are not unreasonably large, meaning that they don't generate extreme univariate outliers in some of the variables, and (2) they are genuine outliers, meaning that the observations for the 5% contaminated individuals completely fail to satisfy the moment conditions of the bulk of the data. As argued in the introduction, such a situation is not unlikely to occur in practice due to data errors or the approximate nature of the postulated model. Similar simulation designs of the present type are often used for evaluating the properties of estimators in situations with outliers, see, e.g., Rousseeuw and Leroy (1987) and Hoek et al. (1995).

Using the different combinations of the parameter values, we generate panels with and without outliers. The number of Monte-Carlo replications is 2,500. The simulations were run on a Sun-Sparc workstation. Computing the nonrobust and the robust GMM estimator took about 3.2 and 4.9 CPU seconds, respectively, such that the computational burden of the robust estimator is not prohibitively large compared to that of its nonrobust counterpart. The increase in computation time is mainly due to the use of the S estimator. The simulation results are presented in Table IV.

We first focus on the samples without outliers. These are presented under the heading *clean*. The estimates of  $\alpha$  and  $\beta$  are quite accurate on the average, both for the robust and nonrobust GMM procedure. Exceptions are the estimates of  $\alpha$  for  $\alpha = 0.8$  and  $\sigma_*^2 = 2$ , which have biases of about 17%. Except in the case  $\alpha = 0.8$  and  $\sigma_*^2 = 2$  for the robust estimator, the mean of the standard error obtained from an estimate of the asymptotic covariance matrix of the GMM estimator (in parentheses), lies close to the Monte-Carlo estimate of the standard deviation (in brackets) for both estimators. This implies that the use of asymptotic theory in the present setting is reasonably accurate for assessing the variability of the estimates. A last thing to note is the higher standard

TABLE I  
 ROBUST AND NONROBUST GMM ESTIMATES FOR A SIMPLE DYNAMIC MODEL  
 WITH THE AHN-SCHMIDT INSTRUMENT MATRIX

$\sigma_*^2$	$\alpha$	Nonrobust GMM				Robust GMM			
		Clean		Contaminated		Clean		Contaminated	
		$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\beta}$
2	0.0	-0.016	1.008	0.012	0.730	-0.015	1.007	-0.019	1.008
		(0.041)	(0.053)	(0.044)	(0.055)	(0.041)	(0.055)	(0.043)	(0.057)
		[0.045]	[0.056]	[0.092]	[0.113]	[0.049]	[0.060]	[0.048]	[0.059]
	0.4	0.367	0.609	0.215	0.467	0.367	0.608	0.357	0.611
		(0.050)	(0.049)	(0.049)	(0.051)	(0.050)	(0.051)	(0.054)	(0.055)
		[0.053]	[0.053]	[0.110]	[0.109]	[0.061]	[0.054]	[0.060]	[0.055]
0.8	0.663	0.199	0.151	0.134	0.659	0.199	0.641	0.194	
	(0.083)	(0.106)	(0.061)	(0.110)	(0.077)	(0.110)	(0.081)	(0.113)	
	[0.094]	[0.113]	[0.184]	[0.246]	[0.109]	[0.120]	[0.113]	[0.122]	
8	0.0	-0.005	1.003	0.058	0.740	-0.003	1.003	-0.005	1.003
		(0.022)	(0.028)	(0.030)	(0.035)	(0.022)	(0.028)	(0.022)	(0.029)
		[0.023]	[0.029]	[0.069]	[0.094]	[0.025]	[0.031]	[0.025]	[0.032]
	0.4	0.393	0.604	0.322	0.489	0.393	0.603	0.391	0.604
		(0.023)	(0.025)	(0.031)	(0.031)	(0.025)	(0.026)	(0.026)	(0.027)
		[0.025]	[0.026]	[0.076]	[0.085]	[0.027]	[0.029]	[0.027]	[0.029]
0.8	0.776	0.204	0.245	0.197	0.774	0.205	0.771	0.204	
	(0.036)	(0.021)	(0.040)	(0.030)	(0.038)	(0.022)	(0.039)	(0.022)	
	[0.039]	[0.022]	[0.168]	[0.085]	[0.044]	[0.024]	[0.043]	[0.024]	

NOTE: The symbol  $\bar{\alpha}$  denotes the Monte-Carlo mean of the estimates of the autoregressive coefficient  $\alpha$ . Between parentheses is the Monte-Carlo mean of the estimated standard error, which is based on the asymptotic normality of the GMM estimator. In square brackets is the Monte-Carlo standard deviation of the estimates of  $\alpha$ . Similar definitions hold for the coefficient  $\beta$  of the exogenous variable. The number of Monte-Carlo replications is 2500.

error of the robust GMM estimates compared to nonrobust GMM. This is due to the fact that the traditional GMM estimator is optimal in the present setting. The robust GMM is consistent, but less efficient, because it assigns smaller weights to some of the (non-outlying) observations. This efficiency loss is the ‘insurance premium’ one pays in order to obtain robustness, see Section 1.

The reduced sensitivity of the robust GMM estimator to outliers is illustrated by the next set of simulations. These are presented under the heading *contaminated*. Except for the estimate of  $\alpha$  for  $\alpha = 0.0$  and  $\sigma_*^2 = 2$ , the robust GMM estimates are closer to their true values than the traditional GMM estimates. Especially for large values of  $\alpha$ , the nonrobust GMM estimator demonstrates a clear bias towards zero. A similar effect is found if the ordinary least squares estimator is used for autoregressive processes with outliers, see, e.g., Hoek, Lucas, and van Dijk (1995). Table IV also reveals that the variance of both estimators increases if outliers are present in the data, albeit that

TABLE II  
 SIZE OF THE ROBUST AND NONROBUST OVER-IDENTIFYING  
 RESTRICTIONS TEST

$\sigma_*^2$	$\alpha$	Nonrobust GMM				Robust GMM			
		Clean		Contaminated		Clean		Contaminated	
		5%	10%	5%	10%	5%	10%	5%	10%
2	0.0	4.4	9.0	100.0	100.0	3.9	9.5	4.0	8.3
	0.4	3.9	9.3	100.0	100.0	4.8	9.8	3.1	7.3
	0.8	6.9	12.8	100.0	100.0	5.7	11.2	6.0	11.5
8	0.0	4.4	9.3	100.0	100.0	3.3	7.9	4.8	9.9
	0.4	4.8	9.1	100.0	100.0	4.2	8.7	4.2	8.7
	0.8	4.7	9.8	100.0	100.0	4.4	10.0	5.4	10.7

NOTE: The entries in the table are the number of rejections of the over-identifying restrictions test using the critical value from the  $\chi^2$  distribution with 43 degrees-of-freedom and a level of five and ten per cent, respectively. The number of Monte-Carlo replications is 2500. Standard errors of the entries are thus at most five per cent.

the variance of the nonrobust estimator increases far more than that of the robust estimator. For the sample size used in the simulations, the approximation to the finite sample distribution of the estimators using the asymptotic normality is much better for the robust GMM estimator than for the nonrobust estimator, at least if the data contain outliers. This can be seen by comparing the Monte-Carlo standard deviation of the estimates (in brackets) with the Monte-Carlo mean of the standard error based on the normal approximation (in parentheses). The standard errors based on asymptotic normality grossly understate the true variability of the nonrobust GMM estimator, while there are almost no discrepancies between the two measures of variability for the robust GMM estimator.

We now turn to the results of the over-identifying restrictions test. This test is discussed in Hansen (1982) and can be used to test whether the moment conditions are satisfied. The test is asymptotically  $\chi^2$  distributed. The degrees of freedom parameter equals the number of moment conditions less the number of parameters. In our case, this results in 43 degrees of freedom. For each of the Monte-Carlo simulations, we calculate whether the test rejects at the 5% and 10% level. The number of rejections is presented in Table II.

For samples without outliers, the robust and nonrobust tests have approximately the correct size. If outliers are added to the sample, the size of the robust test remains roughly constant. By contrast, the standard, nonrobust test breaks down completely. It always rejects the null hypothesis that the moment conditions are satisfied. This dramatic result is brought about by contaminating only 5% of the cross-sectional units in such a way that no extreme univariate outliers are generated. The results for the test based on the robust GMM estimator are better than those of the nonrobust estimator,

because the robust estimator automatically assigns less weight to the contaminated observations.

We also performed additional simulation experiments in order to investigate the sensitivity of the robust and nonrobust GMM estimator to other deviations from Assumption 1 than outliers. The conclusions are reported here, and the actual results are available upon request. Four additional experiments were conducted: (1) Student  $t(3)$  instead of normally distributed errors; (2) 5% of the individuals have the same DGP but with parameters  $\tilde{\alpha} = -\alpha$  and  $\tilde{\beta} = -1 + \alpha$ ; (3) errors  $\varepsilon_{it}$  follow a typical GARCH(1,1) process with parameters 0.8 for lagged variance and 0.15 for lagged squared innovations; (4)  $\varepsilon_{it}$  have variance proportional to  $\exp(x_{it}(1 - \rho^2)^{1/2}/\sigma_\xi)$ , such that there is cross-sectional heteroskedasticity as well as heteroskedasticity over time. The experiment with  $t(3)$  errors resulted in results very similar to the ones reported in Table IV. For the GARCH simulations and the simulations with 5% mild contamination (experiment 2), the performance of the robust and nonrobust GMM estimator is virtually identical. For cross-sectional heteroskedasticity as well as heteroskedasticity over time (experiment 4), the robust estimator has a slightly better performance in terms of efficiency. In all experiments, however, both the robust and nonrobust GMM estimates have the same value on average as for the uncontaminated samples in Table IV. We conclude that the robust and nonrobust estimators have approximately the same performance if there are no real outliers in the sample. If there are such outliers, however, a large bias may show up in the traditional nonrobust GMM estimates, whereas a much smaller bias enters the robust GMM estimates. This makes our outlier robust GMM estimator a useful diagnostic tool in empirical exercises to assess whether the results obtained with the traditional GMM estimator are driven by a few aberrant observations. Such observations may be analyzed using the robust GMM weights  $\phi_{itN}$  which are available after estimation.

## 5. DETERMINANTS OF CAPITAL STRUCTURE: EMPIRICAL RESULTS

Since the famous paper by Modigliani and Miller (1958) many theories have been developed to explain patterns in leverage policies. There is an abundance of theoretical literature in this field. We refer to the survey articles of Ravid (1988) and Harris and Raviv (1991). Although much empirical research has been done on capital structure determinants, it is a common belief that empirical research is still behind its theoretical counterpart, see, among others, Rajan and Zingales (1995). Moreover, the empirical capital structure studies give divergent results as to what are the determinants of leverage, see also Harris and Raviv (1991, Table IV).

Our research approach differs from most capital structure determinants studies in three ways. First, we incorporate the dynamic behavior of leverage in our model. Myers (1984) and Fischer et al. (1989) stress the importance of deviations from optimal capital structures and the existence of dynamic patterns in leverage. Second, we use an outlier robust estimation method. There are good reasons to use an outlier robust estimator in the present context: corporate finance data at the individual firm level

may easily display aberrant behavior due to, e.g., recording errors, sectoral differences, measurement problems following from the use of different accounting systems, and the approximate character of the postulated model itself. The lack of outlier robustness in previous studies may have severe implications for the validity of the statistical conclusions. Finally, we use a panel data set, whereas most empirical finance studies are based on time series averages of cross-sectional parameter estimates. A panel data analysis has some well-known advantages: more observations are used, multi-collinearity problems are mitigated, and the problem of omitted, time-constant variables can be controlled for in a natural way. The only study which employs a model similar to ours is Shuetrim et al. (1993). They use an Australian data set.

Our analysis uses annual balance sheet data of American non-financial firms. The data are obtained from the autumn 1993 edition of Datastream International. We selected a balanced panel of 715 companies with six complete annual observations of leverage between 1987 and 1992. The first year is used for the lagged endogenous variable. We also consider a reduced sample consisting of 518 firms and obtained after deleting univariate outliers from the complete sample. In this way we mimic a preliminary data scanning procedure commonly used for individual firm data.

The model is expressed in (1). For both samples, we compute the traditional, non-robust GMM estimator and its outlier robust extension. This gives us four sets of parameter estimates. The robust GMM estimator is implemented using the specifications for  $\psi$  and  $\psi_t$  as described in Section 3. For the instrument matrix, we use the format of Ahn and Schmidt, as described in (5). It appears that the robust GMM estimator is in most cases stable across the different samples, whereas the traditional, nonrobust estimator is not. Moreover, the over-identifying restrictions test supports the moment conditions used by the robust GMM estimator, but rejects the moment conditions used by the nonrobust estimator.

The remainder of the application is organized as follows. Subsection 5.1 considers the choice of variables and Subsection 5.2 presents the results.

### 5.1 *Determinants and Proxy Variables*

The dependent variable, i.e., leverage, equals the ratio of a firm's book value of total debt to the book values of equity and total debt. The determinants we use to explain the differences in debt levels, namely: economic expansion, size, profitability, liquidity, and non-debt tax shields.<sup>7</sup>

Firms that face economic expansion and that have many investment opportunities are sensitive to agency costs due to moral hazard problems. Three types of problems are well known: asset substitution (Jensen and Meckling, 1976), under-investment (Myers, 1977), and over-investment (Jensen, 1986). All three theories suggest a negative effect of growth opportunities on leverage. By contrast, the pecking order theory of Myers (1984) suggests a positive relation: firms in rapidly expanding industries are more likely

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<sup>7</sup>The working version of this paper contains further details on the construction of the variables.

to raise more debt than firms in stable industries. We use the realized relative difference in sales between two consecutive years to measure current economic expansion.

There are several reasons to consider size as a potential determinant of capital structure. First, the transaction costs depend on the size of the firm, see Smith (1977) and Marsh (1982). Second, the risks of larger firms are more diversified. This decreases the expected (relative) bankruptcy costs of debt, see Ang et al. (1982). Third, the degree of asymmetric information may be size dependent, which is important for the amount of agency costs. We control for size effects in two ways. First, we scale all variables (except the variable for size) by total assets. Second, we include the number of employees as an explanatory variable.

There are several reasons to expect an effect of profitability on leverage. First, for a given level of operation risk, the consequences of the financial risks of debt are less severe for more profitable firms. Second, the terms of debt contracts often depend on the current state of the firm. Third, profitability affects the amount of internal equity affecting the reliance on other capital sources. We use earnings before tax and interest payments as a proxy for profitability.

We now consider the effect of the firm's liquidity on leverage. The probability of financial distress depends on the match between the distribution of the firm's obligations and the distribution of the cash flows. The expected bankruptcy costs are less as the firm's has a good liquidity position which may lead to positive relation between liquidity and the amount of debt. We use the current ratio to measure liquidity, i.e., the ratio of current assets to total assets.

Modigliani and Miller (1963) argued that the tax shield function of debt affects the value of a firm. The value of the tax shields created by interest payments is influenced by the amount of non-debt tax shields, e.g, depreciation. See also Titman and Wessels (1988). If taxes are important for the capital structure, we expect a negative effect of depreciation on leverage.

## 5.2 *Results*

Table III shows the robust and nonrobust estimates for the complete sample of 715 firms. All estimated coefficients are significantly different from zero. Moreover, the estimated coefficient of the lagged leverage variable is between zero and one. Contradicting several moral hazard theories, growing firms adopt a higher debt-to-equity ratio. Larger firms have relatively more debt in their capital structure. This size effect is commonly found in related studies. Profitability and liquidity are negatively related to the debt-to-equity ratio. This is consistent with the bankruptcy costs hypothesis. Finally, depreciation has a negative influence on leverage as suggested by the non-debt tax shield hypothesis. An interesting question is whether the moment conditions used to identify the parameters are valid. The over-identifying restrictions test of Hansen (1982), which has 104 degrees-of-freedom, indicates a violation of the underlying moment conditions at the 1% level.

Using the robust GMM estimator for the complete sample, all estimated parameters

TABLE III  
ROBUST AND NONROBUST GMM ESTIMATES

	N=715		N=518	
	Nonrobust	Robust	Nonrobust	Robust
Lagged Leverage	0.451 (0.036)*	0.482 (0.032)*	0.399 (0.039)*	0.400 (0.028)*
Growth in Sales	0.024 (0.010)	0.142 (0.037)*	0.211 (0.070)*	0.146 (0.044)*
ln(Employees)	0.029 (0.006)*	0.031 (0.011)*	0.007 (0.012)	0.037 (0.010)*
Earnings	-0.110 (0.048)*	-3.681 (0.296)*	-3.160 (0.416)*	-3.652 (0.329)*
Liquidity	-0.165 (0.049)*	-0.547 (0.052)*	-0.471 (0.068)*	-0.552 (0.059)*
Depreciation	-0.138 (0.038)*	-0.537 (0.138)*	-0.232 (0.164)	-1.02 (0.167)*
Over-identifying Restrictions Test	2 46 [0.00]	1 24 [0.08]	2 33 [0.00]	1 03 [0.51]

NOTE: Standard errors are in parentheses and p-values in brackets. The symbol \* denotes significance at the 5% level.

are significant with the same sign as the nonrobust estimates. Despite the similarities between the nonrobust and robust estimates, the magnitudes of the parameter estimates differ considerably. These differences motivate an examination of the observation weights used by the robust estimator.

The robust estimator uses unit weights for 1879 observations, it partially down-weights 132 observations, and it disregards 849 observations. So about 30% of all observations are ignored by the robust estimation procedure. Considering the large reduction of the sample from 715 to 518 firms by deleting only large univariate outliers (see below), this rejection percentage can be expected. Statistical support to use the reduced set of observations is given by the over-identifying restrictions test. In contrast to the nonrobust over-identifying restrictions test, the robust test does not suggest a violation of the moment conditions at the 5% level.

We also compute the standard, nonrobust GMM estimator for a cleaned sample. This sample is obtained from the first sample by deleting firms that have univariate outliers in one or more of the variables. We consider a firm to be an outlier if it has a value for the  $k$ th regressor outside the interval  $(m_k - 4d_k, m_k + 4d_k)$ , where  $m_k$  and  $d_k$

are the median and the scaled median absolute deviation (MAD) of the  $k$ th regressor, respectively (see Hampel et al. (1986) for a definition of the MAD). Due to this deletion procedure, our sample reduces to 518 firms. The reduction leads to two insignificant parameters estimates: the estimated parameters of the depreciation variable and the size variable have become insignificant. The over-identifying restrictions test indicates again a violation of the traditional moment conditions at the 1% level.

It is illuminating to investigate the causes of the differences between the results of the robust estimator applied to the complete sample and the results of the nonrobust estimator applied to the set of 518 firms. Before we can give an economic explanation for the different results, we have to present some statistical facts. In total there are 2860 leverage observations. The univariate outlier detection method and the robust GMM estimator regard the same set of 1762 observations as fully informative. They both fully disregard the same set of 625 observations. Roughly stated, the exclusion is mainly due to the presence of univariate outliers in the instrument matrix. There is a very small set of 46 observations that are partially weighted by the robust method and ignored by the nonrobust method. There are now three groups of undiscussed observations left. The first set includes 117 observations that are ignored by the nonrobust estimation approach, but which are still considered as fully informative by the robust estimator. This set is rather small (4%). This can be considered as a confirmation that the robust estimator is capable to identify univariate outliers. The second set includes 86 observations that are down-weighted by the robust GMM estimator, but fully used by the standard nonrobust approach. The third set consists of 224 observations that are completely disregarded in the robust estimation procedure, but used by the traditional approach. The observations in the second and third set (about 11% of all observations) may explain the differences in the two sets of parameter estimates and the over-identifying restrictions tests.

There are two principal reasons why the above 310 observations are ignored by the robust estimator, but used by the traditional approach. First, the traditional approach is not capable to identify multivariate outliers in the space of instruments. This explains the zero weights of 154 out of the 310 observations. Second, the traditional method is not capable to correct for special events that are not captured by the model, i.e., large residuals. The robust estimator accounts for these events. A variety of causes for unusual shocks in the debt-equity ratio can be given, for example, large new investments or debt-equity swaps motivated by macroeconomic considerations. Deriving a model that allows for all these issues is impossible. An investigation of the estimation results reveals that large time-series variations in leverage explain for a large part the zero weights of the remaining 156 observations. About 50% of the 156 observations are related to an absolute change in leverage between two years of more than 0.20. For the complete sample, only 8 per cent of the observations are related to such large changes in leverage.

Finally, we discuss the last column of Table III, which shows the robust GMM estimates for the reduced sample. These estimates are similar to the outlier robust estimates obtained with the complete panel, except the magnitude of the parameter estimate of the depreciation variable. The over-identifying restrictions test does not

indicate a violation of the robust moment conditions at the 5% level. This suggests that the robust GMM estimator can, at least partly, be used as a substitute for an extensive preliminary data scanning stage.

## 6 CONCLUDING REMARKS

We have studied the generalized methods of moments (GMM) estimator for a linear dynamic panel data model with error components. Econometric models are at best an approximation to the complex process called reality. Therefore, it is not surprising that a postulated model may easily fit most, but not all of the observed data. Using the the influence function and Monte-Carlo simulations, we have shown that the standard GMM estimator used in the literature is very sensitive to aberrant observations.

We proposed an outlier robust variant of the GMM estimator that is less sensitive to aberrant observations. Although the robust estimator was proposed in the context of linear dynamic panel data models, it can be applied in the more general context where one has linear moment conditions. The main principle underlying the robust estimator is the replacement of standard moment conditions by observation weighted moment conditions, where aberrant observations automatically receive less weight. A standard statistical asymptotic theory for the estimator was obtained.

The robustness of the new estimator was illustrated using simulations and an empirical application. If the data contain no outliers, the robust GMM estimator is somewhat less efficient than standard GMM. This is the price one has to pay in order to be protected against the adverse effects of outliers and aberrant observations. For the settings studied in this paper, however, the efficiency loss is quite small. If outliers are present, the robust GMM estimator is much less biased than its traditional, nonrobust counterpart. The outlier robust estimator, therefore, is a useful tool to the econometrician's toolkit. It can be used to check whether results obtained with traditional nonrobust GMM estimators are driven by a few aberrant observations. The robust estimator produces observation weights that can be used to diagnose the cause of the differences and to indicate routes for model re-specification.

In the empirical application, we investigated the determinants of the firm's capital structure. The results indicate markedly different results between the traditional GMM estimator and the robust estimator. Upon closer inspection of the observation weights produced by the robust estimator, we see that most of the differences are caused by the presence of univariate outliers in the regressors. If these outliers are removed, the robust and nonrobust estimation results are much closer, though still not identical. This may be due to multivariate outliers and/or leptokurtosis and adds a further dimension to empirical analyses of the capital structure puzzle.

## APPENDIX A.

In this appendix we give a description of the data. We use annual balance sheet data of American non-financial firms. The data are obtained from Datastream International.

Out of the 991 firms we initially selected, 276 firms have missing values for the period 1987–1992. These firms are discarded, resulting in a balanced panel of 715 companies with six complete annual observations of leverage between 1987 and 1992. The first year is used for the lagged endogenous variable. Datastream International classifies these firms in the following industry groups (the number of firms in a particular industry are given between parentheses): electronics and electrical equipment (157), oil exploration & production (104), food manufacturers (79), leisure and hotels (62), metallurgy and steel (43), retailers (73), packing and paper (44), general chemicals (56), media publishing (29), motor vehicles (16) motor components (37), manufacturing (7), and mail order stores (8).

The definitions used in constructing the variables are as follows. The ratio that measures *leverage*, is based on item 733 of Datastream. This item provides the ratio of the sum of the book values of subordinated debt, total loan capital and borrowing repayable within one year, over the sum of the book values of equity and capital reserves and total deferred tax minus intangible assets. Since the sample distribution of this ratio is very skewed, we transform it to the ratio debt over total assets. The transformation is given by  $(1 + x^{-1})^{-1}$  where  $x$  is the debt over equity ratio. The transformed ratio is taken as our final measure of leverage. *Growth* is the annual growth rate in the amount of sales of goods and services to third parties, relating to the normal activities of the company (constructed from item 104). *Profitability* is measured as the net profit derived from normal trading activities before depreciation and operating provisions (item 135). *Non-debt tax shields* is measured by depreciation, representing provisions for amounts written off, and depreciation of fixed assets and assets leased in (item 136). This variable includes amounts written off intangibles, but excludes amortization of deferred charges. The *size* of a firm is measured by the total number of domestic and overseas employees, including part-time, when available (item 219). *Liquidity* is equal to the ratio of current assets (item 376) to the sum of current assets and current liabilities (item 389). Item 389 includes current provisions, creditors, borrowings repayable within one year, and other current liabilities. Both the non-debt tax shield variable and the profitability variable are divided by total assets (item 339), which includes the net total of land and buildings, plant and machinery, construction in progress and any other fixed assets. In order to remove the apparent skewness of several variables, we subjected these variables to a logarithmic transformation, possibly after adding a constant if the original variable could become nonpositive.

## APPENDIX B

In this appendix we prove the appropriateness of the moment conditions in (7) for the robust GMM estimator. We also prove the consistency and asymptotic normality of the estimator for  $T$  fixed and  $N \rightarrow \infty$ . The notation is that of Section 3. Apart from assuming that the model is correctly specified and that Assumption 1 is satisfied, we assume the following.

**Assumption 2.** (1) The vectors  $\{p_i\}_{i=1}^N$  are independently and identically distributed random vectors that have a density of the form

$$|V|^{-1/2} f((p-m)'V^{-1}(p-m)),$$

with  $f$  a nonincreasing function, defined on the interval  $[0, \infty)$  and  $V$  a positive definite matrix. Also,  $f$  and the function  $\rho$ , which is used to define the S estimator (see Section 3), have a common point of decrease, i.e., there exists a constant  $c_1$ , such that  $f(c_0) > f(c_1) > f(c_2)$  and  $\rho(c_0) > \rho(c_1) > \rho(c_2)$  for  $c_2 > c_1 > c_0 > 0$ .

- (2)  $E(W_i \Phi_i D_T (y_i - Z_i \gamma)) \neq 0$  for all  $\gamma \in \Gamma \setminus \{\gamma_0\}$ .
- (3)  $\psi_t$  is a bounded continuously differentiable function with derivative  $\psi'_t$ . The function  $\psi$  is bounded, anti-symmetric, and twice continuously differentiable with first and second order derivative equal to  $\psi'$  and  $\psi''$ , respectively.
- (4) Let  $\gamma \in \Gamma$ ,  $\sigma \in \Sigma$ ,  $m \in \mathcal{M}$ , and  $V \in \mathcal{V}$ , with  $\Gamma \subset \mathbf{R}^{K+1}$ ,  $\Sigma \subset (0, \infty)$ , and  $\mathcal{M} \subset \mathbf{R}^{T(K+1)+1}$ , and  $\mathcal{V}$  a subset of the space of positive definite matrices.  $\Gamma \times \Sigma \times \mathcal{M} \times \mathcal{V}$  is compact.

The sequence  $\{\hat{\sigma}_N\} \in \Sigma$  converges almost surely to  $\sigma_0$ , while the sequence of weight matrices for the moment conditions  $\{A_N\}$  converges almost surely to a positive definite matrix  $A_0$ .

- (5) Let  $\theta = (\sigma, m', \text{vec}(V)')$ .

$$E[\partial(W_i \Phi_i D_T \varepsilon_i) / \partial \gamma'] \text{ has full column rank, and}$$

$$|E[(\partial(W_i \Phi_i D_T \varepsilon_i) / \partial \gamma')(\partial(W_i \Phi_i D_T \varepsilon_i) / \partial \gamma')']| \neq 0.$$

Finally we assume that, where needed, the order of integration and differentiation may be interchanged. Moreover, all expectations that are used in the proof are assumed to be finite. A discussion of the assumptions above is found in Subsection 3.3. Part 1 of Assumption 2 can be relaxed without affecting the consistency of the robust GMM estimator. In fact, we only need that the estimate of  $m$  converges to a vector of finite constants and that the estimate of  $V$  converges to a finite positive definite matrix, both at rate  $N^{1/2}$ . For simplicity, however, we stick to the assumption as stated in part 1. For extensions, a good starting reference is Sakata and White (1996).

**PROOF OF LEMMA 1:** Given the specification in (6), we obtain that the moment conditions in (7) can be written as

$$(B1) \quad E \left( \sum_{t=1}^{T-1} v_t(w_{it}) \cdot w_{it} \cdot \psi((\varepsilon_{i,t+1} - \varepsilon_{it})/\sigma) \right).$$

Using Assumption 1, the anti-symmetry of  $\psi(\cdot)$ , and the fact that  $w_{it} \in I_{it}$ , we obtain

$$\begin{aligned} E \left( \sum_{t=1}^{T-1} v_t(w_{it}) \cdot w_{it} \cdot E(\psi((\varepsilon_{i,t+1} - \varepsilon_{it})/\sigma) | I_{it}) \right) &= \\ E \left( \sum_{t=1}^{T-1} v_t(w_{it}) \cdot w_{it} \cdot E(\psi((\varepsilon_{it} - \varepsilon_{i,t+1})/\sigma) | I_{it}) \right) &= \\ -E \left( \sum_{t=1}^{T-1} v_t(w_{it}) \cdot w_{it} \cdot \psi((\varepsilon_{i,t+1} - \varepsilon_{it})/\sigma) \right) &, \end{aligned}$$

such that (7) holds.

*Q.E.D.*

We now proceed with proving the consistency and asymptotic normality of the robust GMM estimator.

**PROOF OF THEOREM 1:** It follows from Assumption 2 and the linearity of model (1), that Assumptions 1 to 3 of Gallant (1987, Chapter 3) hold.

Assumption 8 of Gallant is also easily checked. The assumption of correct model specification trivially implies the convergence of Gallant's sequence  $\{\gamma_n^0\}$ . Furthermore, the parts 1 and 3 of Assumption 2 imply the conditions stated in Davies (1987). Therefore, the S estimator  $(\hat{m}, \hat{V})$  is strongly consistent and converges at the rate  $N^{-1/2}$ . From Assumption 1 and the anti-symmetry of  $\psi(\cdot)$  we have (compare the proof of Lemma 1)

$$\begin{aligned} E \left[ \frac{\partial W_i \Phi_i D_T \varepsilon_i}{\partial \theta'} \right] &= \left( -E \left[ \sum_{t=1}^{T-1} \tilde{w}_{it} \psi' \left( \frac{e_{it}^0}{\sigma} \right) \frac{e_{it}^0}{\sigma^2} \right], E \left[ \sum_{t=1}^{T-1} \frac{\partial \tilde{w}_{it}}{\partial (m', \text{vec}(V)')} \psi \left( \frac{e_{it}^0}{\sigma} \right) \right] \right) \\ \text{(B2)} \quad &= 0, \end{aligned}$$

with  $\theta$  as defined in part 5,  $e_{it}^0 = \varepsilon_{i,t+1} - \varepsilon_{it}$ , and  $\tilde{w}_{it} = v_t(w_{it})w_{it}$ . Note that the last equality uses the fact that  $\psi'(\cdot)$  is symmetric. Together with parts 2 and 5 of Assumption 2, these results imply Gallant's Assumptions 6 and 8. Note that (B2) implies a type of block-diagonality of the information matrix between the regression parameters  $\gamma$  and the nuisance parameters  $m$  and  $V$ , see also the remark following Theorem 1.

Assumption 9 of Gallant can be replaced by the interchangeability of differentiation and integration. Finally, Gallant's Assumption 10 is trivially fulfilled, while his Assumption 11 can be checked using (B2).

We can now apply Theorems 7 and 9 of Gallant (1987, Chapter 3) and conclude that the robust GMM estimator is strongly consistent and asymptotically normal. *Q.E.D.*

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TABLE IV  
 ROBUST AND NONROBUST GMM ESTIMATES FOR A SIMPLE DYNAMIC MODEL  
 WITH THE AHN-SCHMIDT INSTRUMENT MATRIX

$\sigma_*^2$	$\alpha$	Nonrobust GMM				Robust GMM			
		Clean		Contaminated		Clean		Contaminated	
		$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\alpha}$	$\bar{\beta}$
	0.0	-0.016	1.008	0.012	0.730	-0.015	1.007	-0.019	1.008
		(0.041)	(0.053)	(0.044)	(0.055)	(0.041)	(0.055)	(0.043)	(0.057)
		[0.045]	[0.056]	[0.092]	[0.113]	[0.049]	[0.060]	[0.048]	[0.059]
2	0.4	0.367	0.609	0.215	0.467	0.367	0.608	0.357	0.611
		(0.050)	(0.049)	(0.049)	(0.051)	(0.050)	(0.051)	(0.054)	(0.055)
		[0.053]	[0.053]	[0.110]	[0.109]	[0.061]	[0.054]	[0.060]	[0.055]
	0.8	0.663	0.199	0.151	0.134	0.659	0.199	0.641	0.194
		(0.083)	(0.106)	(0.061)	(0.110)	(0.077)	(0.110)	(0.081)	(0.113)
		[0.094]	[0.113]	[0.184]	[0.246]	[0.109]	[0.120]	[0.113]	[0.122]
	0.0	-0.005	1.003	0.058	0.740	-0.003	1.003	-0.005	1.003
		(0.022)	(0.028)	(0.030)	(0.035)	(0.022)	(0.028)	(0.022)	(0.029)
		[0.023]	[0.029]	[0.069]	[0.094]	[0.025]	[0.031]	[0.025]	[0.032]
8	0.4	0.393	0.604	0.322	0.489	0.393	0.603	0.391	0.604
		(0.023)	(0.025)	(0.031)	(0.031)	(0.025)	(0.026)	(0.026)	(0.027)
		[0.025]	[0.026]	[0.076]	[0.085]	[0.027]	[0.029]	[0.027]	[0.029]
	0.8	0.776	0.204	0.245	0.197	0.774	0.205	0.771	0.204
		(0.036)	(0.021)	(0.040)	(0.030)	(0.038)	(0.022)	(0.039)	(0.022)
		[0.039]	[0.022]	[0.168]	[0.085]	[0.044]	[0.024]	[0.043]	[0.024]

NOTE: The symbol  $\bar{\alpha}$  denotes the Monte-Carlo mean of the estimates of the autoregressive coefficient  $\alpha$ . Between parentheses is the Monte-Carlo mean of the estimated standard error, which is based on the asymptotic normality of the GMM estimator. In square brackets is the Monte-Carlo standard deviation of the estimates of  $\alpha$ . Similar definitions hold for the coefficient  $\beta$  of the exogenous variable. The number of Monte-Carlo replications is 2500.